**Exam time: 3 hours**

**Instructions:**
*1. For writing your answers use both sides of the paper in the answer booklet.*

*2. Additional sheets taken, if any, should be properly attached to the main answer booklet.*

*3. Maximum time is 3 hours and Maximum Possible Score is 100*

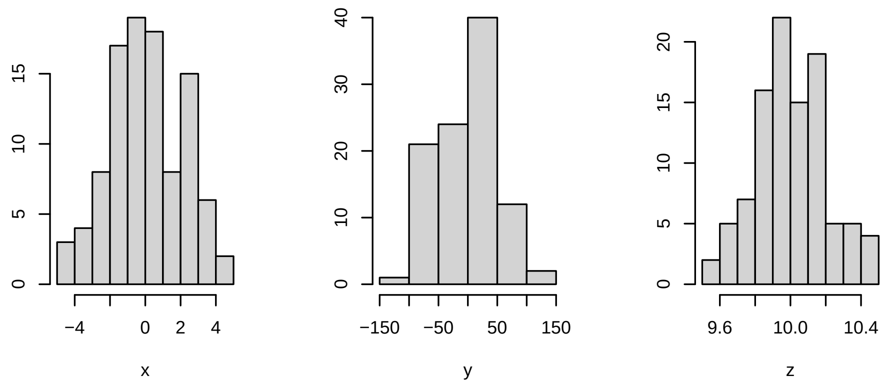
**Score**

| Q.No. | Alloted Score | Score |
|---|---|---|
| 1. | (10 points) | |
| 2. | (15 points) | |
| 3. | (15 points) | |
| 4. | (20 points) | |
| 5. | (20 points) | |
| 6. | (15 points) | |
| 7. | (10 points) | |
| Total | 105 | |

**Number of Extra sheets attached to the answer script:** ____________

1. In each of the following below, please circle the correct choice. No justification is required.

(A): Consider the three histograms given below of datasets x, y and z




The ordering of the dataset from the smallest to biggest standard deviations is given by:

$(i)$(x, y, z) $(ii)$(x, z, y) $(iii)$(y, x, z) $(iv)$(y, z, x) $(v)$(z, y, x) $(vi)$(z, x, y)

(B): Suppose we are given $X_1, X_2, \dots X_n$ i.i.d sample from a population $X$. Suppose an estimate for $\theta$ is given by $\hat{\theta} = g(X_1, X_2, \dots X_n)$ and $\hat{\theta}_i^\star = g_{(i)}(X_1, X_2 \dots, X_{i-1}, X_{i+1}, \dots X_n)$ for $i = 1, \dots n$. Then the jackknife bias is given by:

(i) $(n-1)\left(\frac{1}{n}\sum_{i=1}^{n}\hat{\theta}_i^\star - \hat{\theta}\right)$ (ii) $\left(\frac{1}{n}\sum_{i=1}^{n}\hat{\theta}_i^\star - \hat{\theta}\right)$ (iii) $\min\{\mid \hat{\theta}_i^\star - \hat{\theta} \mid\}$

2. (15 points) At the ISI co-rec basketball league in the 10 games played team *Unit-disc* scored:

$$59, 62, 59, 74, 70, 61, 62, 66, 62, 75$$

Assume that the number of points scored by *Unit-disc* is Normally distributed.

(a) Compute a 95% confidence interval for the mean, $\mu$.

(b) We want to test the null hypothesis that $\mu = 63$ versus the alternative hypothesis that $\mu \neq 63$. Decide and execute a test that can check if there is enough evidence whether one can reject the null hypothesis at 5% level of significance.

3. (15 points) Let $X_1, X_2, X_3, \ldots, X_n$ of i.i.d. radioactive measurements following Exponential($\theta$). Find the maximum likelihood estimate for $\theta$.

4 (20 points)The following `R` code simulates a random variable `X`

```
> i = 0
> T=0
> Sum = 0
> while (T<10) {
+     T = T+1
+     U = runif(1, min=0, max =1)
+     Y = -log(U)
+     Sum = Sum +Y
+     i = Sum
+ }
> X = i
```

Find the distribution of $X$.

5. (20 points) Shyamala is interested in studying average years of schooling in India. They hypothesized that the mean years of school for people 18 years old or above is higher than 8.5 years. They drew a sample of 9 from 2011 census and find out that the mean number of years of schooling for the sample is 8.515, with a SD of 4.5. We wish to test

$$H_0 : \mu = 8.5 \text{ versus } H_A : \mu \neq 8.5$$

   (a) Compute the $T$-statistic.

   (b) Find the $p$-value.

   (c) Describe the meaning of $p$-value and what inference can you draw from it if your level of significance is 0.005

6. (15 points) The student body at an undergraduate university is 20% Masters, 24% third years, 26% Second year, and 30% first year students. Suppose a researcher takes a sample of 50 such students. Within the sample there are 12 Masters, 14 Third years, 10 Second years, and 14 First years. The researcher claims that his sampling procedure should have produced independent selections from the student body, with each student equally likely to be selected. Is this a plausible claim given the observed results?

7. (10 points) $X$ and $Y$ are two independent samples from populations with distributions $F$ and $G$ respectively. We want to test $F = G$. Suppose,

$$X : 53, 38, 69$$

and

$$Y : 44, 40, 61$$

Compute the Mann-Whitney U test statistic (discussed in class).

**June 4th, 2026** **Name (Please Print)** ______________________

**Statistical Tables for Reference if Needed.**

**Table 1 :** in $(i, j)$-th entry provides $\mathbb{P}(t_i \leq j)$, with $t_i \sim$ t-distribution with $i$ degrees of freedom.

| | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 | 1.1 | 1.2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 0.570 | 0.604 | 0.636 | 0.667 | 0.695 | 0.722 | 0.746 | 0.768 | 0.789 | 0.807 | 0.823 |
| 3 | 0.573 | 0.608 | 0.642 | 0.674 | 0.705 | 0.733 | 0.759 | 0.783 | 0.804 | 0.824 | 0.842 |
| 4 | 0.574 | 0.610 | 0.645 | 0.678 | 0.710 | 0.739 | 0.766 | 0.790 | 0.813 | 0.833 | 0.852 |
| 5 | 0.575 | 0.612 | 0.647 | 0.681 | 0.713 | 0.742 | 0.770 | 0.795 | 0.818 | 0.839 | 0.858 |
| 6 | 0.576 | 0.613 | 0.648 | 0.683 | 0.715 | 0.745 | 0.773 | 0.799 | 0.822 | 0.843 | 0.862 |
| 7 | 0.576 | 0.614 | 0.649 | 0.684 | 0.716 | 0.747 | 0.775 | 0.801 | 0.825 | 0.846 | 0.865 |
| 8 | 0.577 | 0.614 | 0.650 | 0.685 | 0.717 | 0.748 | 0.777 | 0.803 | 0.827 | 0.848 | 0.868 |
| 9 | 0.577 | 0.615 | 0.651 | 0.685 | 0.718 | 0.749 | 0.778 | 0.804 | 0.828 | 0.850 | 0.870 |

**Table 2 :** in $(i, j)$-th entry provides $\mathbb{P}(t_i \leq j)$, with $t_i \sim$ t-distribution with $i$ degrees of freedom.

| | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 1.9 | 2.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.750 | 0.765 | 0.779 | 0.791 | 0.803 | 0.813 | 0.822 | 0.831 | 0.839 | 0.846 | 0.852 |
| 2 | 0.789 | 0.807 | 0.823 | 0.838 | 0.852 | 0.864 | 0.875 | 0.884 | 0.893 | 0.901 | 0.908 |
| 3 | 0.804 | 0.824 | 0.842 | 0.858 | 0.872 | 0.885 | 0.896 | 0.906 | 0.915 | 0.923 | 0.930 |
| 4 | 0.813 | 0.833 | 0.852 | 0.868 | 0.883 | 0.896 | 0.908 | 0.918 | 0.927 | 0.935 | 0.942 |
| 5 | 0.818 | 0.839 | 0.858 | 0.875 | 0.890 | 0.903 | 0.915 | 0.925 | 0.934 | 0.942 | 0.949 |
| 6 | 0.822 | 0.843 | 0.862 | 0.879 | 0.894 | 0.908 | 0.920 | 0.930 | 0.939 | 0.947 | 0.954 |
| 7 | 0.825 | 0.846 | 0.865 | 0.883 | 0.898 | 0.911 | 0.923 | 0.934 | 0.943 | 0.950 | 0.957 |
| 8 | 0.827 | 0.848 | 0.868 | 0.885 | 0.900 | 0.914 | 0.926 | 0.936 | 0.945 | 0.953 | 0.960 |
| 9 | 0.828 | 0.850 | 0.870 | 0.887 | 0.902 | 0.916 | 0.928 | 0.938 | 0.947 | 0.955 | 0.962 |
| 10 | 0.830 | 0.851 | 0.871 | 0.889 | 0.904 | 0.918 | 0.930 | 0.940 | 0.949 | 0.957 | 0.963 |

**Table 3 :** in $(i, j)$-th entry provides $\mathbb{P}(t_i \leq j)$, with $t_i \sim$ t-distribution with $i$ degrees of freedom.

| | 2.20 | 2.22 | 2.24 | 2.26 | 2.28 | 2.30 | 2.32 | 2.34 | 2.36 | 2.38 | 2.40 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 0.921 | 0.922 | 0.923 | 0.924 | 0.925 | 0.926 | 0.927 | 0.928 | 0.929 | 0.930 | 0.931 |
| 3 | 0.942 | 0.943 | 0.945 | 0.946 | 0.947 | 0.948 | 0.948 | 0.949 | 0.950 | 0.951 | 0.952 |
| 4 | 0.954 | 0.955 | 0.956 | 0.957 | 0.958 | 0.959 | 0.959 | 0.960 | 0.961 | 0.962 | 0.963 |
| 5 | 0.960 | 0.961 | 0.962 | 0.963 | 0.964 | 0.965 | 0.966 | 0.967 | 0.968 | 0.968 | 0.969 |
| 6 | 0.965 | 0.966 | 0.967 | 0.968 | 0.969 | 0.969 | 0.970 | 0.971 | 0.972 | 0.973 | 0.973 |
| 7 | 0.968 | 0.969 | 0.970 | 0.971 | 0.972 | 0.973 | 0.973 | 0.974 | 0.975 | 0.976 | 0.976 |
| 8 | 0.971 | 0.971 | 0.972 | 0.973 | 0.974 | 0.975 | 0.976 | 0.976 | 0.977 | 0.978 | 0.978 |
| 9 | 0.972 | 0.973 | 0.974 | 0.975 | 0.976 | 0.977 | 0.977 | 0.978 | 0.979 | 0.979 | 0.980 |

**Table 4:** in $(i, j)$-th entry provides $\mathbb{P}(Z \le i + j)$, with $Z \sim \text{Normal}(0, 1)$

| | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | 0.10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 | 0.5398 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 | 0.5793 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 | 0.6179 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 | 0.6554 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 | 0.6915 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 | 0.7257 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 | 0.7580 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 | 0.7881 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 | 0.8159 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 | 0.8413 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 | 0.8643 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 | 0.8849 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 | 0.9032 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 | 0.9192 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 | 0.9332 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 | 0.9452 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 | 0.9554 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 | 0.9641 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 | 0.9713 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 | 0.9772 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 | 0.9821 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 | 0.9861 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 | 0.9893 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 | 0.9918 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 | 0.9938 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 | 0.9953 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 | 0.9965 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 | 0.9987 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 | 0.9998 |

**Table 5:** in $(i, j)$-th entry provides $\mathbb{P}(\chi_i^2 \leq j)$, with $\chi_i^2 \sim$ Chi-square with $i$ degrees of freedom.

| | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 0.181 | 0.221 | 0.259 | 0.295 | 0.330 | 0.362 | 0.393 | 0.423 | 0.451 | 0.478 | 0.503 |
| 3 | 0.060 | 0.081 | 0.104 | 0.127 | 0.151 | 0.175 | 0.199 | 0.223 | 0.247 | 0.271 | 0.294 |
| 4 | 0.018 | 0.026 | 0.037 | 0.049 | 0.062 | 0.075 | 0.090 | 0.106 | 0.122 | 0.139 | 0.156 |
| 5 | 0.005 | 0.008 | 0.012 | 0.017 | 0.023 | 0.030 | 0.037 | 0.046 | 0.055 | 0.065 | 0.076 |
| 6 | 0.001 | 0.002 | 0.004 | 0.006 | 0.008 | 0.011 | 0.014 | 0.018 | 0.023 | 0.028 | 0.034 |
| 7 | 0.000 | 0.001 | 0.001 | 0.002 | 0.003 | 0.004 | 0.005 | 0.007 | 0.009 | 0.012 | 0.014 |
| 8 | 0.000 | 0.000 | 0.000 | 0.000 | 0.001 | 0.001 | 0.002 | 0.002 | 0.003 | 0.004 | 0.006 |
| 9 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.001 | 0.001 | 0.001 | 0.002 | 0.002 |

**Table 6:** in $(i, j)$-th entry provides $\mathbb{P}(\chi_i^2 \leq j)$, with $\chi_i^2 \sim$ Chi-square with $i$ degrees of freedom.

| | 1.5 | 1.6 | 1.7 | 1.8 | 1.9 | 2.0 | 2.1 | 2.2 | 2.3 | 2.4 | 2.5 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 0.528 | 0.551 | 0.573 | 0.593 | 0.613 | 0.632 | 0.650 | 0.667 | 0.683 | 0.699 | 0.713 |
| 3 | 0.318 | 0.341 | 0.363 | 0.385 | 0.407 | 0.428 | 0.448 | 0.468 | 0.487 | 0.506 | 0.525 |
| 4 | 0.173 | 0.191 | 0.209 | 0.228 | 0.246 | 0.264 | 0.283 | 0.301 | 0.319 | 0.337 | 0.355 |
| 5 | 0.087 | 0.099 | 0.111 | 0.124 | 0.137 | 0.151 | 0.165 | 0.179 | 0.194 | 0.209 | 0.224 |
| 6 | 0.041 | 0.047 | 0.055 | 0.063 | 0.071 | 0.080 | 0.090 | 0.100 | 0.110 | 0.121 | 0.132 |
| 7 | 0.018 | 0.021 | 0.025 | 0.030 | 0.035 | 0.040 | 0.046 | 0.052 | 0.059 | 0.066 | 0.073 |
| 8 | 0.007 | 0.009 | 0.011 | 0.013 | 0.016 | 0.019 | 0.022 | 0.026 | 0.030 | 0.034 | 0.038 |
| 9 | 0.003 | 0.004 | 0.005 | 0.006 | 0.007 | 0.009 | 0.010 | 0.012 | 0.014 | 0.017 | 0.019 |

**Table 7:** in $(i, j)$-th entry provides $\mathbb{P}(\chi_i^2 \leq j)$, with $\chi_i^2 \sim$ Chi-square with $i$ degrees of freedom.

| | 2.50 | 2.52 | 2.54 | 2.56 | 2.58 | 2.60 | 2.62 | 2.64 | 2.66 | 2.68 | 2.70 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 0.713 | 0.716 | 0.719 | 0.722 | 0.725 | 0.727 | 0.730 | 0.733 | 0.736 | 0.738 | 0.741 |
| 3 | 0.525 | 0.528 | 0.532 | 0.535 | 0.539 | 0.543 | 0.546 | 0.549 | 0.553 | 0.556 | 0.560 |
| 4 | 0.355 | 0.359 | 0.363 | 0.366 | 0.370 | 0.373 | 0.377 | 0.380 | 0.384 | 0.387 | 0.391 |
| 5 | 0.224 | 0.227 | 0.230 | 0.233 | 0.236 | 0.239 | 0.242 | 0.245 | 0.248 | 0.251 | 0.254 |
| 6 | 0.132 | 0.134 | 0.136 | 0.138 | 0.141 | 0.143 | 0.145 | 0.148 | 0.150 | 0.152 | 0.155 |
| 7 | 0.073 | 0.074 | 0.076 | 0.077 | 0.079 | 0.081 | 0.082 | 0.084 | 0.085 | 0.087 | 0.089 |
| 8 | 0.038 | 0.039 | 0.040 | 0.041 | 0.042 | 0.043 | 0.044 | 0.045 | 0.046 | 0.047 | 0.048 |
| 9 | 0.019 | 0.020 | 0.020 | 0.021 | 0.021 | 0.022 | 0.023 | 0.023 | 0.024 | 0.024 | 0.025 |